Exercise 27

Find the limit or show that it does not exist.

$$\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$$

Solution

Multiply the numerator and denominator by the complex conjugate.

$$\begin{aligned}
\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right) &= \lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)\cdot\frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x}\\
&= \lim_{x\to\infty}\frac{\left(\sqrt{9x^2+x}-3x\right)\left(\sqrt{9x^2+x}+3x\right)}{\sqrt{9x^2+x}+3x}\\
&= \lim_{x\to\infty}\frac{(9x^2+x)-(9x^2)}{\sqrt{9x^2+x}+3x}\\
&= \lim_{x\to\infty}\frac{x}{\sqrt{9x^2+x}+3x}\\
&= \lim_{x\to\infty}\frac{x}{\sqrt{x^2\left(9+\frac{1}{x}\right)}+3x}\\
&= \lim_{x\to\infty}\frac{x}{x\sqrt{9+\frac{1}{x}}+3x}\\
&= \lim_{x\to\infty}\frac{x}{x\left(\sqrt{9+\frac{1}{x}}+3\right)}\\
&= \lim_{x\to\infty}\frac{1}{\sqrt{9+\frac{1}{x}}+3}\\
&= \frac{\lim_{x\to\infty}1}{\lim_{x\to\infty}\left(\sqrt{9+\dfrac{1}{x}}+3\right)}\\
&= \frac{\lim_{x\to\infty}1}{\lim_{x\to\infty}\sqrt{9+\dfrac{1}{x}}+\lim_{x\to\infty}3}\\
&= \frac{1}{\sqrt{9+0}+3}\\
&= \frac{1}{6}
\end{aligned}$$